Phase Noise Measurements

Convert Time Interval Error Into Phase Noise

The short term stability of an oscillator is characterized by measuring jitter in the time domain and phase noise in the frequency domain. Since both testing methods characterize the same underlying phenomena it is possible to relate the two. Also, since LeCroy oscilloscopes incorporate analysis tools such as the time domain based Jitter and Timing Analysis (JTA) and the frequency domain based Fast Fourier Transform (FFT), it is possible to make both measurements within a single instrument.

Phase noise is a random modulation of the phase of an oscillator's output due to a variety of internal processes within the oscillator circuit. This random modulation manifests itself in the time domain as jitter, the random displacement of clock edges from a nominal position. In the frequency domain, phase noise is seen as a broadening of the oscillator signal's frequency spectrum due to modulation sidebands.

A typical phase noise measurement setup is show in figure 1. The output from the oscillator under test is mixed with the output of a low phase noise reference oscillator set to the same frequency with a relative phase of 90°. The phase shift is adjusted to exact phase quadrature indicated by a minimum DC level at the output of the mixer. The mixer is now operating as a phase detector and produces a voltage proportional to the phase difference of the two sources. Since the reference oscillator has a very low phase noise the mixer output is principally a function of the phase noise of the oscillator under test. This assumes that neither oscillator has any significant amplitude modulation.

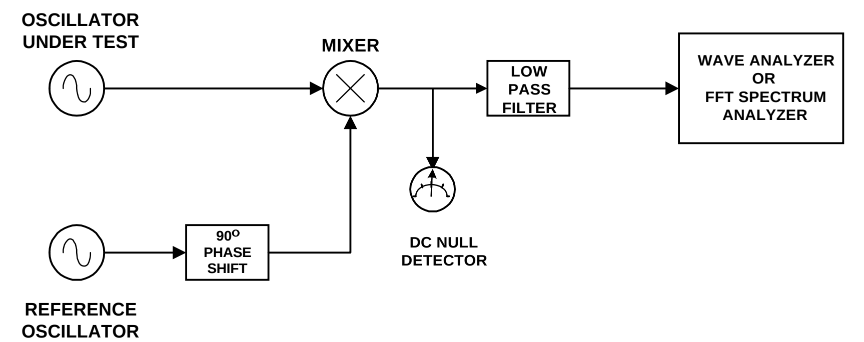

Figure 1– A typical test setup for measuring phase noise

The output of the mixer is low pass filtered to remove the higher frequency sum terms and mixer leakage spectral components. The output spectrum is read using a wave analyzer or FFT spectrum analyzer. The phase noise is usually displayed as a power spectral density using units of deciBels relative to the carrier power per unit Hz bandwidth (dBc/ Hz)

The time interval error (TIE) function in the LeCroy JTA option returns the time or unit interval difference between the time an input signal crosses a preset voltage threshold and the ideal location of a user specified reference frequency. The TIE function is plotted as time or unit interval vs. time. If TIE is displayed using time difference the plot graphically shows the phase modulation envelope. By suitable scaling and FFT analysis it is possible to display a phase noise plot based on the measured TIE.

The single sideband phase noise with respect to the carrier at an offset frequency f_m, $\alpha(f_m)$,can be estimated as:

$$\alpha(fm) = 20 \log_{10} (\varphi_d/2)$$

Where : φ_d is the peak phase deviation in radians

The peak phase deviation in terms of the FFT spectrum of time interval error, $\Delta t(f_m)$is :

$$\varphi_d = 2\pi f_c \, \Delta t(f_m)$$

Inserting this into the expression for the phase noise we get:

$$\alpha(fm) = 20 \log_{10} (2\pi f_c \, \Delta t(f_m)/2)$$

Re-computing this in terms of available oscilloscope math operations:

$$\alpha(fm) = 20 \log_{10} (\pi f_c) + 20 \log_{10} (\Delta t(f_m))$$

A system calibration, based upon the measurement of a phase modulated carrier with known phase deviation was used to verify these calculations in practice. The signal source is a LeCroy LW420 generating a 10 MHz Carrier phase modulated at 10 kHz with a peak phase deviation of 10 degrees (0.174 radians). An FFT analysis of the signal is shown in figure 2.

The sideband amplitude based on the relationship:

Single sideband to carrier ratio (dB) =
$$20 \log_{10} (\varphi_d/2)$$

Where: φ_d is the phase deviation in radians

For a phase deviation of 10° (0.174 radians) we expect a sideband level of:

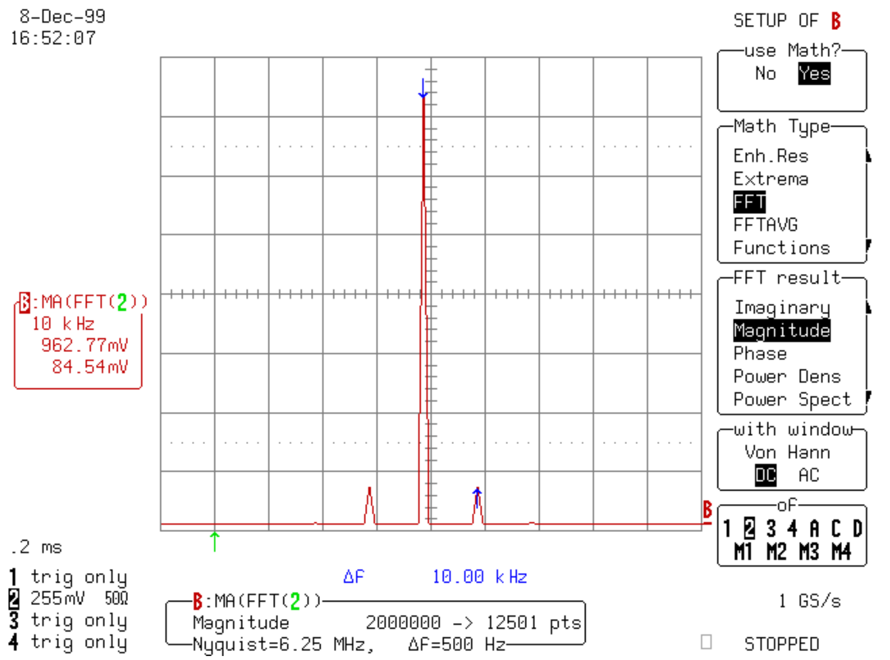

Figure 2 - The verification of the sideband amplitudes of a 10 MHz phase modulated signal for a 10 degree phase deviation at a modulation frequency of 10 kHz

Single sideband to carrier ratio (dB_c)
$$= 20 \log_{10} (\varphi_d/2)$$
$$= 20 \log_{10} (0.174/2) = -21.21 \text{ dB}_c$$

Based on the measured sideband (V_m = 84.54 mV)and carrier amplitudes (Vc= 962.77 mV), shown in figure 2, the measured sideband amplitude is

Single sideband to carrier ratio (dB_c)
$$= 20 \log_{10} (Vm / Vc)$$
$$= 20 \log_{10} (84.54 / 962.77) = -21.13 \text{ dB}_c$$

This measurement confirms the expected level of the modulation sideband and this value will be used to confirm the phase noise measurement. The same signal was applied to the oscilloscope and the configuration was changed to calculate phase noise using the FFT of the JTA time interval error function. The FFT was then scaled using the values calculated using the equation:

$$\alpha(fm) = 20 \log_{10} (\pi f_c) + 20 \log_{10} (\Delta t(f_m))$$

For the carrier frequency (f**c**) of 10 MHz

$$\alpha(fm) =$$
$$20 \log_{10} (\pi 10^7) + 20 \log_{10} (\Delta t(f_m)) =$$
$$150 + 20 \log_{10} (\Delta t(f_m))$$

This equation relates the single sideband phase noise, α(fm), to the FFT of the time interval error, $\Delta t(f_m)$. The phase noise is computed by performing the FFT on the TIE function, weighting the FFT logarithmically, and then rescaling the result. The multiplicative constant, 20, converts the units into dB. The additive constant, 150, scales to data values to dBc (dB relative to carrier).

The screen image in figure 3 shows the results of a phase noise measurement using this technique to analyze the signal measured previously. The result shows a sideband level of -21.19 dBc in the bottom trace (trace D). The Zoom+Math menu summarizes the set up of each of the 4 math traces. Starting with the TIE function of channel 2 in Trace A, the FFT of TIE in trace B, the log of trace B in trace C (not shown), and the rescaling of trace C.

The phase noise function in trace D of figure 3 has a dynamic range of approximately 70-75 dBc. This can be extended by using FFT averaging. This requires adding another math function. The FFT average is stored in one of the storage memories. The log and rescale functions are computed on the stored trace.

Figure 4 shows a phase noise measurement using FFT averaging to extend the dynamic range of the measurement. The modulation frequency in this example is 100 kHz and the phase deviation is 0.31 x 10^{-3} radians, This corresponds to a sideband level of -76dBc. Note that the scope reading is -75.1 dBc. The baseline for this measurement is -80 dBc which is equivalent to a peak phase deviation of 3 ps. This is a 6 ps peak-peak jitter or about a 1 ps rms jitter, assuming a Gaussian distribution.

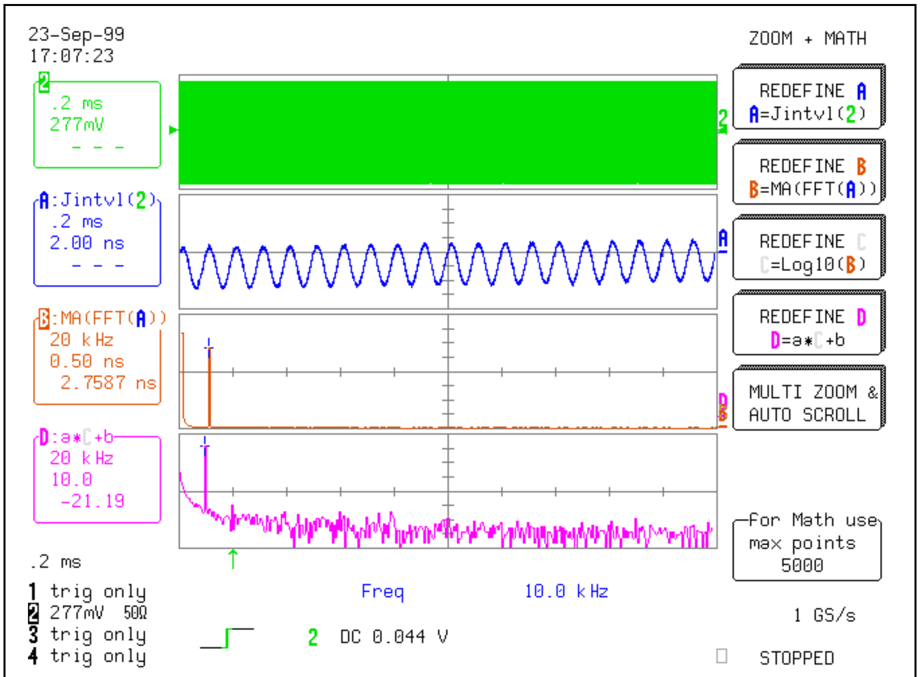

Figure 3 - Measuring the phase noise of a signal with a known phase deviation using the FFT of the time interval error.

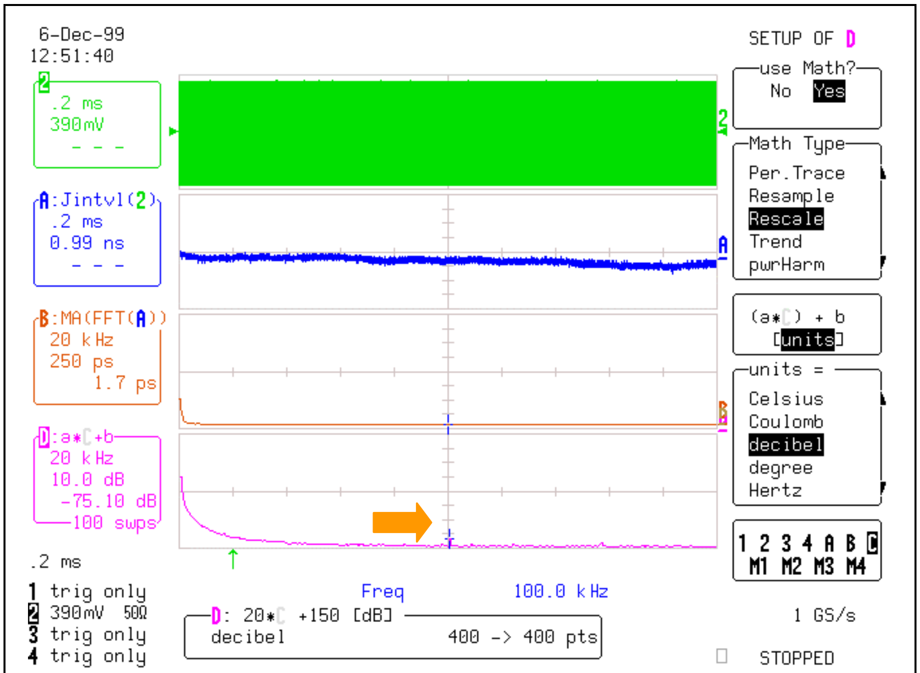

Figure 4 - Using FFT averaging to extend the dynamic range of the measurement. Arrow indicates 100 kHz modulation peak

Note also that the rescale function has been used to re-label the units in trace D to dB.

The accompanying table shows the linearity of this measurement for known phase deviations of a 100 kHz modulation on a 10 MHz carrier. Measured values track expected values within 1 dB over the 80 dB dynamic range. The signal source for this measurement was an HP 8648C externally modulated by a LeCroy LW420.

Table 1, Measured Phase Noise vs. Theoretically Expected Value		
Phase Deviation (radians)	Theoretical Sideband Level (dBc)	Measured Sideband Level (dBc)
0.1	-26.02	-25.69
0.08	-27.95	-27.64
0.05	-32.04	-31.68
0.03	-36.48	-37.57
0.01	-46.02	-46.89
0.005	-52.04	-52.95
0.0025	-58.06	-58.96
0.00125	-64.08	-64.66
0.000625	-70.1	-70.25
0.0003175	-76.12	-75.1

The vertical scale of the phase noise analysis can also be changed to power spectral density, in dBc/Hz, by dividing the spectrum by the noise bandwidth of the FFT analysis. The equation for phase noise, in dBc/Hz, as a function of time jitter becomes:

$$\alpha(fm) = 20 \log_{10}(\pi f_c) + 20 \log_{10}(\Delta t(f_m)) - 10 \log_{10}(\Delta f \text{ ENBW})$$

Where:
Δf is the resolution bandwidth of the FFT(found in the FFT setup menu) and **ENBW** is the effective noise bandwidth of the selected weighting function found in the operators manual

Figure 5 shows an example of a measurement of phase noise in units of power spectral density (dBc/Hz) . The signal is a 10 MHz carrier phase modulated with a peak phase deviation of 0.1 radians at 100 KHz. The resolution bandwidth of the FFT is 1 KHz using rectangular weighting with an ENBW of 1.00.

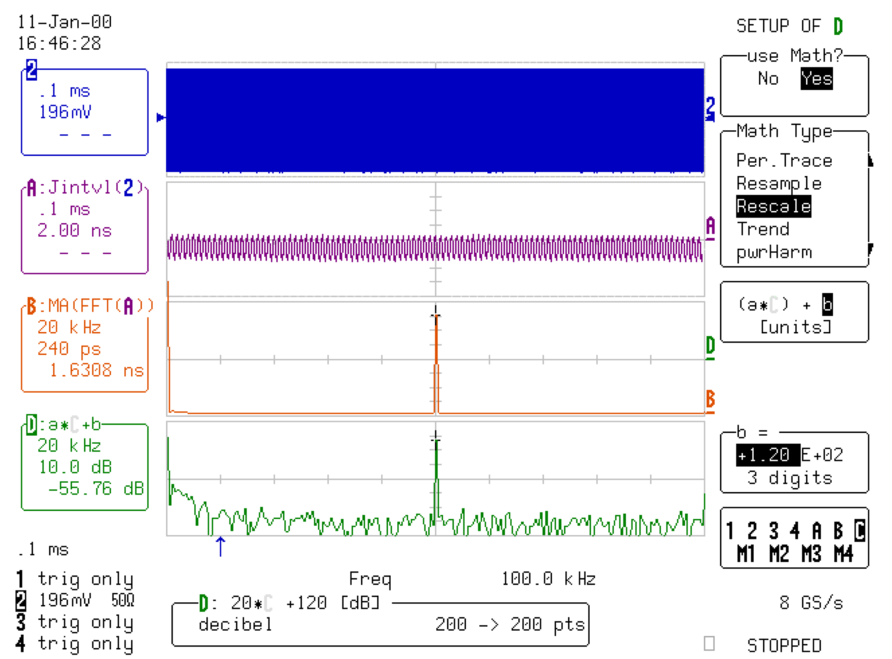

Figure 5 - The phase noise of a signal with a known phase deviation displayed as power spectral density in units of dBc/Hz

In Summary, it is possible to calculate the phase noise of a signal based upon the measured time jitter. The calculations can be carried out simultaneously with time domain jitter measurements. The dynamic range for signals that are not averaged is about –70dBc. Averaging in the frequency domain, increases the dynamic range to 80 dBc . The measurement of phase noise makes use of the basic jitter measurement functions supported by chained math operations. These features are hallmarks of LeCroy digital oscilloscopes

Reference :
“Frequency Synthesizers, Theory and Design” by Vadim Mannassewitsch, John Wiley and Sons copyright 1976